Your task will be to code a simulation of image compression based on the approximate low rank structure of the set of image patches.

You will write functions

- \( C = \text{my\_block\_transform}(I, B); \)
- \( I = \text{my\_block\_untransform}(C, B, m, n); \)

The function \texttt{my\_block\_transform} takes as input an \( m \times n \) grayscale image \( I \), extracts all the distinct \( 8 \times 8 \) blocks from the image using the matlab command \texttt{im2col}, and finds the representation of each block in the \((64 \times 64)\) orthogonal basis \( B \). The output \( C \) is a \( 64 \times n\text{blocks} \) array which gives the coefficients for each block, where \( n\text{blocks} \) is the total number of blocks from the image.

The function \texttt{my\_block\_untransform} takes as input an array \( C \), a \( 64 \times 64 \) basis \( B \), and output size parameters \( m \) and \( n \). It outputs an \( m \times n \) image \( I \) reconstructed from the coefficients in \( J \); you may use the function \texttt{col2im} if you find it convenient.

For example, suppose \( I \) is a \( 24 \times 24 \) image. We can write \( I \) in block form:

\[
I = \begin{pmatrix}
I_1 & I_4 & I_7 \\
I_2 & I_5 & I_8 \\
I_3 & I_6 & I_9
\end{pmatrix},
\]

where \( I_k \) is a \( 8 \times 8 \) matrix for \( k \in \{1, 2, \ldots, 9\} \). Using \texttt{im2col} on \( I \), we obtain

\[
J = (J_1 \ J_2 \ J_3 \ J_4 \ J_5 \ J_6 \ J_7 \ J_8 \ J_9) = \text{im2col}(I, [8 \ 8], \text{'distinct'}),
\]

where \( J_k \) is a \( 64 \times 1 \) column vector. If we write each column \( J_k \) of \( J \) in the \( 64 \times 64 \) basis \( B \), we get the matrix

\[
C = (C_1 \ C_2 \ C_3 \ C_4 \ C_5 \ C_6 \ C_7 \ C_8 \ C_9).
\]

You will then be provided with a training data set consisting of 200000 \( 8 \times 8 \) grayscale patches of images. You will need to reshape these into
a 64 × 200000 matrix \( X \), and construct a basis \( B \) using the left singular vectors of \( X \). You will use these, and your code from this assignment more in the next homework assignment, but play with this \( B \) a bit. What is it like? how is it different from a basis of the form \texttt{orth(randn(64))}? how are the columns of \( J \) different with the two bases but the same image?