**Thermodynamics (2010.3)**
Reference:
Any physical chemistry text book
Look in your favorite Biochemistry; Physical Chemistry, or Organic Chemistry Books for the sections on Thermodynamics & the section on pK_a’s.
The Gennis chapter handouts.

Pay most attention to pages 4-8 of this handout.

**Energy conversion**

1 J = 1 kg/m/s^2 = 1 CV (coulomb volt) = 1 Nm (newton meter)
1 cal heats 1 g of water from 14.5 to 15.5°
1 cal = 4.186 J
1 kcal = 4.186 kJ (1 food calorie is a kcal)
1 V = 23 kcal
1 V = 8000 cm^-1 (Ask yourself how cm^-1 can be a unit of energy).
N = 6.023 x 10^{23} objects (1 mole)
C = 6.241 x 10^{18} electrons (1 coulomb)
F = N electrons = 96,494 C/mol = 96494/V/mol (Faraday)
273.15 K = 0°C
k_b = 1.3807 x 10^{-23} J/K (Boltzmann's constant)
R = N k_b = 1.987 cal/K/mol = 8.3145 J/K/mol (gas constant)
Systems are:
Can exchange
<table>
<thead>
<tr>
<th></th>
<th>Material</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Isolated</strong></td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td><strong>Closed</strong></td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Open</strong></td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

In an open system you really can't solve any thermodynamic equations exactly because you don't get to equilibrium. But you can tell
- How far a reaction is from equilibrium.
- What the equilibrium mixture of reactants and products would be if the system is closed and allowed to come to equilibrium.
- The maximum work the system can do as it moves towards equilibrium. This equals the minimum energy needed to put the system into its current state.

**Energy** (E) is a macroscopic, extensive property. Energy is a state function. Defines:
1) If a system can do work.
2) There is a unique amount of energy for a given state
3) $\Delta E$ between 2 states is independent of the path
4) Energy is conserved (first law of thermodynamics)

2nd law
- What is possible is not fully defined by the first law.
  - You can't cool a cold reservoir to make work. This would use the energy of the heat-moving heat from a colder to hotter place and extracting some of this energy to do work. You can do work and move energy from a cold place to a hotter one (e.g. your refrigerator).
  - You can't transfer heat (Q) from one reservoir to another to extract energy for work (W) if both reservoirs are at the same temperature.
  - You can always use work to make heat!
**Free energy** - Tells us which direction a process goes in?

A spontaneous process does not just increase entropy or decrease enthalpy. It's the balance of the two that counts. If the universe were to optimize only \( S \) or \( H \) what would equilibrium look like? Equilibrium is a balance between the conflicting requirements to maximize entropy and minimize energy. For example, stronger bonds are at lower (more favorable) enthalpy but fix the molecule more tightly reducing (making less favorable) the entropy.

**Gibbs free energy-state function**

\[
\Delta G = \Delta H - T\Delta S = q_p - T\Delta S
\]

\[
dG = -SdT + Vdp + \sum_{j=1}^{M} \mu_i dN_i \quad (\mu_i \text{ is the chemical potential})
\]

\( E = W + Q \)

(the change in energy going from state 1 to 2 can be contributed by **work** (done on or done by) and by **heat** (put in or out))

work - changes in energy that involve coherent (non-random) molecular movement.
change in volume; (mechanical)
moving across a concentration gradient (chemiosmotic work-)
electrons moving between 2 different potentials (electrochemical work)

heat - result of random motions; heat always flows from higher to lower temperature.

\( H = E + PV \) **Enthalpy**

Under conditions of constant pressure (changes in volume can yield work) then \( \Delta H \) is the absorption of heat at constant pressure
\( \Delta H = \Delta Q_p \) If no other work done

\( W = P\Delta V + w' \) (w' is non-PV work)

\( \Delta H = \Delta E + p\Delta V = q_p - w + P\Delta V = q_p + w' \)

measure in constant pressure calorimeter where w'= 0

**Entropy**

Is a measure of how a system can be arranged. All other things being equal (eg if all arrangements have the same enthalpy) the more ways the better.

\( S = k_b \ln W \)

\( \Delta S \geq \frac{q}{T} \) (heat transferred by a process at constant \( T \)) (dissolving salts)

\( T\Delta S \geq q_p \)

How many ways can be arrange 4 balls in a box with 9 positions?

\( W = 9 \times 8 \times 7 \times 6 = 126 \) 9 places for 1st; 8 for 2nd; 7 for 3rd; 6 for 4th

\( 4 \times 3 \times 2 \times 1 \) (so \( W = 126; S \approx 2 \text{cal/mol/K ln(126)} = 9.7 \text{ cal/mol/K} \))

\( W = \text{boxes!} \)

objects!

but there are only 4 ways to have all balls next to each other (do you agree?)

\( \Delta G = q_p - T\Delta S - w' \) where \( w' \) is non-PV (Pressure x \( \Delta \)Volume) work

\( TDS \geq q_p \)

\( \Delta G \geq w' \) \( \Delta G \) is the maximum work that can be done
Different forms of energy

Light energy (for $\lambda$ in nm)

$$\Delta G = N_h c = \frac{N h c}{\lambda} = 28667 \text{kcal/mol}/\lambda = 1246.4 \text{eV}/\lambda$$

860 nm $\rightarrow$ 33.33 kcal/mol $\rightarrow$ 1.45 eV

$h = 6.6 \times 10^{-34} \text{J}\cdot\text{sec}$

c = 3 \times 10^8 \text{m/s}$

1 J = 0.24 cal/mol = 2.4 \times 10^{-4} \text{kcal}$

A chemical reaction (see below)

$$\Delta G = \Delta G^\circ + RT \ln [P]/[R]$$

A chemical reaction that involves the transfer of electrons

$$E = E^\circ + \frac{RT}{nF} \ln \frac{D_{ox}}{D_{red}}$$

$$\Delta G^\circ = -nF \Delta E^\circ = -nF \Delta V$$

A concentration gradient across a membrane

$$\Delta G = RT \ln \left( \frac{[\text{in}]}{[\text{out}]} \right) \quad \text{For any molecule - charged or neutral.}$$

The electrical potential across a membrane given the concentration gradient of ions.

$$\Delta V = \Delta \Psi = \frac{RT}{F} \ln \frac{\sum_{\text{cations}} [\text{conc}]_{\text{out}} + \sum_{\text{anions}} [\text{conc}]_{\text{in}}}{\sum_{\text{cations}} [\text{conc}]_{\text{in}} + \sum_{\text{anions}} [\text{conc}]_{\text{out}}}$$

The energy of an ion concentration gradient

$$\Delta G = RT \ln \frac{[\text{conc}]_{\text{in}}}{[\text{conc}]_{\text{out}}} + nF \Delta V$$

Mechanical work

$$W = F \Delta s \quad \text{(force x displacement in the direction of the force)}$$

$$W = \tau \Delta \sigma \quad \text{(torque x angular displacement)}$$

What's the difference between $\Delta G$ and $\Delta G^\circ$?
For a reaction

\[ aA + bB \leftrightarrow cC + dD \]

\[ K_{eq} = \frac{[C]^c[D]^d}{[A]^a[B]^b} \]

ratio of products and reactants at equilibrium

\[ K_{eq} = \exp(-\Delta G^o / RT) = 10^{-\Delta G^o/1.36 \text{kcal} / \text{mole}} \]

\[ \Delta G^o = -(RT)ln K_{eq} = -(2.303RT)\log K_{eq} \]

\[ \Delta G = \Delta G^o + (RT) \ln \frac{[\text{products}]}{[\text{reactants}]} \]

\[ \Delta G = \Delta G^o + (2.303RT) \log \frac{[\text{products}]}{[\text{reactants}]} \]

\[ \Delta G = \Delta G^o + (1.36 \text{kcal} / \text{mole}) \log \frac{[\text{products}]}{[\text{reactants}]} \]

\[ \Delta G = \Delta G^o + (58 \text{meV}) \log \frac{[\text{products}]}{[\text{reactants}]} \]

\[ \Delta G = \Delta G^o + RT \log \frac{[\text{products}]}{[\text{reactants}]} = RT (-\ln K_{eq} + \ln \frac{[\text{products}]}{[\text{reactants}]}) \]

**Now at equilibrium** \[ \frac{[\text{products}]}{[\text{reactants}]} = K_{eq} \] so \[ G = 0 \]

i.e. \[ \Delta G = RT (-\ln K_{eq} + \ln \frac{[\text{products}]}{[\text{reactants}]}) = RT (-\ln K_{eq} + \ln K_{eq}) \]
Two views of $\Delta G^\circ$:

(1) $\Delta G^\circ$ is the energy needed to move the system from equilibrium to the standard state (i.e. the energy of the standard state).

The default standard state has all concentrations at 1 (1 Mole/liter; 1 atmosphere are common). The solvent concentration is for the pure liquid. e.g. water will be at 55.5M.

$$\Delta G = \Delta G^\circ + (1.36 \text{ kcal/mol}) \log \left[ \frac{\text{products}}{\text{reactants}} \right]$$

$$= \Delta G^\circ + 1.36 \text{ kcal/mol} \log \left[ \frac{[1]}{[1]} \right]$$

$$= \Delta G^\circ + 0$$

However, you can define the standard state in any way that is convenient. (see section on acids and bases for how to define a $\Delta G^\circ$ at pH 7 as is standard in many biochemistry tables.

(2) $\Delta G$ at equilibrium is 0; but $\Delta G^\circ= -1.36 \text{ kcal/mol}(\log K_{eq})$. Written in this way $\Delta G^\circ$ tells us about the relative energy of the reactants and products. $\Delta G^\circ$ is positive if there is more reactant in the mixture at equilibrium and negative if there more product.
   
   If a reaction has a positive $\Delta G^\circ$ for $A \rightarrow B$ then it has a negative $\Delta G^\circ$ for $B \rightarrow A$.

If

- $K_{eq} = 0.01$  \quad $\Delta G^\circ = 2*1.36 \text{ kcal/mol} = 2*58 \text{ eV}$
- $K_{eq} = 0.1$  \quad $\Delta G^\circ = 1.36 \text{ kcal/mol} = 58 \text{ eV}$
- $K_{eq} = 1$  \quad $\Delta G^\circ = 0$
- $K_{eq} = 10$  \quad $\Delta G^\circ = -1.36 \text{ kcal/mol} = -58 \text{ eV}$
- $K_{eq} = 100$  \quad $\Delta G^\circ = -2*1.36 \text{ kcal/mol} = -2*58 \text{ eV}$

If $\Delta G^\circ = 2.72 \text{ kcal/mol}$ then at equilibrium $[\text{reactants}]=0.01*[\text{products}]$
If $\Delta G^\circ = 1.36 \text{ kcal/mol}$ then at equilibrium $[\text{reactants}]=0.1*[\text{products}]$
If $\Delta G^\circ = 0$ kcal/mol then at equilibrium $[\text{reactants}]=[\text{products}]$
If $\Delta G^\circ = -1.36 \text{ kcal/mol}$ then at equilibrium $[\text{reactants}]=10*[\text{products}]$
If $\Delta G^\circ = -2.72 \text{ kcal/mol}$ then at equilibrium $[\text{reactants}]=100*[\text{products}]$
For example $\text{ADP} + \text{P}_i = \text{ATP}$ $K_{eq}=10^{-5}$ at pH 7.0

This means

1) The standard free energy of this reaction is:

$$\Delta G^\circ = (-1.36 \text{ kcal/mol}) \log K_{eq} = (-1.36 \text{ kcal/mol}) \log 10^{-5} = 6.8 \text{ kcal/mol}$$

The free energy ($\Delta G$) is 0 when $[\text{P}]/[\text{R}]=10^{-5}$

$$\Delta G = 6.8 + 1.36 \log 10^{-5} = 0 \text{ kcal/mol}$$

2) If you had 1M of ADP and 1M of P$^i$ and 1M of ATP at pH 7 the system would be out of equilibrium. The free energy available to do work is:

$$\Delta G = \Delta G^\circ + (1.36\text{ kcal/mol}) \log[1]/[1][1] = +6.8 \text{ kcal/mol}$$

3) If you have $10^{-3}$M Pi the equilibrium ratio of ATP/ADP = $10^{-7}$ since

$$K_{eq}=10^{-5} = \text{ATP}/(\text{ADP} \times \text{Pi})$$

4) However, ATP/ADP = $10^3$ in the cell. How much work must be done to achieve this concentration ratio (if there is $10^{-3}$M Pi)?

$$\Delta G = \Delta G^\circ + (1.36\text{ kcal/mol}) \log[\text{ATP}]/[\text{ADP}][\text{Pi}]$$

$$= 6.8 \text{ kcal/mol} + 1.36 \log [10^3]/10^{-2} = 13.6 \text{ kcal/mol}$$

We will discuss how the cell finds the energy to maintain this condition so far out of equilibrium when we learn about how the ATPase protein works.

5) If the oxidation/reduction potential for a single electron transfer (at pH 7) $\Delta E^\circ$

Cyt b (ox) +e$^{-} \rightarrow$ Cyt b (red)  -160 mV

Plastocyanin (ox) +e$^{-} \rightarrow$ Plastocyanin (red)  +300 mV

The $\Delta G^\circ=-nF \Delta E^\circ = n \cdot \Delta E^\circ$ (if $\Delta G^\circ$ is in meV and $\Delta E^\circ$ is in mV)

60 meV = 1.36 kcal/mol

PC(red) + Cyt b(ox) $\rightarrow$ PC(ox) + Cyt b(red) $\Delta G^\circ$= - 1$\cdot$ (-160 - (+300)) = +460 meV = +10.4 kcal/mol

Unfavorable to move an electron from PC to Cyt b

Cyt b(red) + PC(ox) $\rightarrow$ Cyt b(ox) + PC(red) $\Delta G^\circ$= - 1$\cdot$ (300 - (-160) ) = -460 meV= +10.4 kcal/mol

6) Mechanical work = $F\cdot$distance;

If I apply a force of 1 N over 1 meter the work done in 1Joule. The homework problem requires care in keeping your exponents correct – nothing more.
The thermodynamic box.

Suppose I have a reaction that

\[
\begin{align*}
A & \leftrightarrow B \\
\Delta G_A^o & \quad \Delta G_B^o \\
\Delta G_{A^*} & \quad \Delta G_{B^*} \\
\cdot A & \leftrightarrow \cdot B \\
\Delta G_{A^*}^o & , \Delta G_{B^*}^o
\end{align*}
\]

Perturb reaction by factor \( \cdot \). What is \( \Delta G_{A^*}^o \)?

\[
\Delta G_{A^*}^o = \Delta G^o - \Delta G_A^o + \Delta G_B^o
\]

This comes from the assumption that the free energy moving around the box eg from A back to A has to be zero.

So if \( A \leftrightarrow \cdot A \) is changed by \( A \text{ kcal/mol} = \Delta G_A^o \)

If \( B \leftrightarrow \cdot B \) is changed by \( B \text{ kcal/mol} = \Delta G_B^o \)

Remember the \( \Delta G_{AB}^o \) for \( A \rightarrow B \) is equal to \(-\Delta G_{BA}^o \quad B \rightarrow A \)