1. Use properties 1° - 7° of determinants listed in our textbook to compute determinants of the following two matrices by row-reducing them. Show work!

\[
A := \begin{bmatrix}
5 & -4 & 1 \\
1 & 1 & -7 \\
0 & 0 & 2
\end{bmatrix}
\]

\[
B := \begin{bmatrix}
3 & 0 & 0 & 0 & 0 \\
1 & 5 & 5 & 1 & -6 \\
0 & 1 & 1 & -7 & -8 \\
0 & 0 & 0 & 0 & -2 \\
0 & 0 & 0 & 2 & 0
\end{bmatrix}
\]

2. Let \(A\) be as in problem (1) and let \(C := \begin{bmatrix}
-4 & 1 \\
1 & -7
\end{bmatrix}\); let \(a_{ij}\) and \(c_{ij}\) denote the entries in \(A\) and \(C\). Recall that for a function \(\sigma\) from the set \(\{1, 2, \ldots, n\}\) to itself, the \(n \times n\) matrix \(P_\sigma\) is the permutation matrix that rearranges the rows of a matrix according to \(\sigma\); that is,\(\]
\[
P_\sigma \begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_n
\end{bmatrix} = \begin{bmatrix}
A_{\sigma(1)} \\
A_{\sigma(2)} \\
\vdots \\
A_{\sigma(n)}
\end{bmatrix}.
\]

(a) List all one-to-one mappings \(\sigma\) from the set \(\{1, 2\}\) to itself, and find the \(2 \times 2\) matrix \(P_\sigma\) for each of them.

(b) Compute \(s_\sigma := c_{1\sigma(1)}c_{2\sigma(2)}d(P_\sigma)\) for each \(\sigma\) in part (a).

(c) Add all the \(s_\sigma\) from part (b).

(d) List all six one-to-one mappings \(\tau\) from the set \(\{1, 2, 3\}\) to itself, and find the \(3 \times 3\) matrix \(P_\tau\) for each of them.

(e) Compute \(t_\tau := a_{1\tau(1)}a_{2\tau(2)}a_{3\tau(3)}d(P_\tau)\) for each \(\tau\) in part (d).

(f) Add the six \(t_\tau\) from part (e); compare to \(d(A)\) you found in problem 1.

(g) Prove carefully that the function given by formula (5.2) on p. 92 of Dym satisfies the three defining properties 1° - 3° of determinants in Theorem 5.1, thereby proving the existence part of that theorem.

3. For each function below, decide which of the properties 1° - 7° of determinants it satisfies. Here, \(d\) is the actual determinant function.

(a) (product of row-sums) \(\alpha(A) := \prod_{i=1}^{n}(\sum_{j=1}^{n} a_{ij})\)

(b) (determinant cubed plus determinant) \(\beta(A) := (d(A))^3 + d(A)\)

As usual, prove your answers!