This problem set is about sections 2.1 and 2.2 of Mathematical Logic Lecture Notes by van den Dries.

1. Let $A := \{p_i : i \in \mathbb{N}\}$, and consider the following subsets of $\text{Prop}(A)$. For each set $\Gamma$, find a model of $\Gamma$ or find a finite subset $\Gamma_0 \subset \Gamma$ that is not satisfiable.

   (a) $\Gamma := \{\leftrightarrow p_i \neg p_{i+1} : i \in \mathbb{N}\}$
   (b) $\Gamma := \{\leftrightarrow \land p_i \neg p_{i+1} \land p_j \neg p_{j+1} : i, j \in \mathbb{N}, i \neq j, j \neq k, i \neq k\}

2. Show that the set $\Gamma_0$ that you found in Problem 1 is inconsistent.

3. Let $X \subset \text{Prop}(A)$ be a set of propositions on $A$. Suppose that $X$ has at least two distinct models. Show that there is some satisfiable set $Y \subset \text{Prop}(A)$ such that $Y \supset X$; or find a counterexample.

If your score on Problem Set 1 was less than 10, you may replace one of the three problems on this problem set by complete, correct solutions to all problems on Problem Set 1.