Math 308: Bridge to Advanced Mathematics

Problems about quantifiers, due in class on Thursday, March 17.

Work on these problems and write down your thoughts, **even if you do not have a complete solution.** Write clearly enough for another student in this course, or for yourself in a year, to understand your work.

1. Fix rational numbers $a$ and $b$.

   (a) Which two of the following four statements mean the same thing?
   
   i. For some rational number $r$, $a = r^3$.
   
   ii. For some rational number $r$, $b = r^3$.
   
   iii. For some rational number $s$, $a = s^3$.
   
   iv. For some rational number $r$, $a = s^3$.

   (b) Which one of those four statements is unclear?

2. (a) Write down two different ways to interpret the ambiguous statement
   “Some positive real number is the square root of any positive integer.”

   (b) Restate the two statements in part (a) using only the following expressions:
   “there exists some”, “for any”, “such that”, “is a real number”, “is an integer”, “≠ 0”, “x”, “y”, “x = y^2”, “if”, “then”, “and”

   (c) Write down the negations of the two statements in part (b).

   (d) For each of the two statements in part (b), prove the statement or prove its negation.

3. (a) Write down two different ways to interpret the ambiguous statement
   “This integer is the square of a rational number; it is an integer.”

   (b) Restate the two statements in part (a) using only the following expressions:
   “there exists some”, “such that”, “is a rational number”, “is an integer”, “≠ 0”, “x”, “y”, “x = y^2”, “if”, “then”, “and”

   (c) Write down the negations of the two statements in part (b).

   (d) (not easy) Prove that the two statements in part (a) are equivalent.

**General homework directions:**

You may discuss homework problems with other students, and I encourage you to do so. However, write your solutions yourself: do not copy them word for word. Acknowledge your collaborators: write on the solutions you hand in "I worked with Jane Lee on problem 3, and with Jose Perez on problems 2a and 4."

If you do not know some of the words used on the homework, look them up before working on the problem: start with our textbooks, then try a dictionary or google. You may also use outside sources, such as books or websites, if you are stuck on a homework problem after at least 30 minutes of thought over at least two days. In any case, provide a traceable reference to your source(s): "wikipedia" or "a theorem in a number theory book" is not traceable; “the wikipedia page for **Equivalence_relation**” and "Theorem 3.7 on p.54 of Burton’s Elementary Number Theory" are traceable.

Failing to acknowledge collaboration or outside sources is called plagiarism; it is a kind of cheating. Cheating is taken very seriously in US colleges. **If I find plagiarism in your problem sets, you will receive no credit and no feedback on problem sets for the rest of the semester.**