A great deal has long been known about student difficulties connecting real-world experiences with what they are learning in their physics classes, making learning basic ideas of classical physics challenging. Understanding these difficulties has led to the development of many instructional approaches that have been shown to help students make connections to the real world, think constructively, and learn the material successfully. However, what happens when making connections to the real world is more complicated. It is one thing to try to figure out how pushing a block with a constant force leads to constant speed, but it is very different to try to build toward an understanding of time dilation. Do the same instructional approaches work here? Also, is it possible that improved instructional approaches lead to improved student approaches when trying to make sense of difficult and very unfamiliar material? In this paper we describe a unique opportunity to perform a controlled experiment by interviewing identical twin brothers working together to resolve the twin paradox. These were intelligent and articulate science students with similar backgrounds but with diverging undergraduate experiences. One happened to take traditional physics classes and the other happened to take classes designed through Physics Education Research.

The twins

We will call the identical twins Fred and George. Both studied science and were intelligent, accomplished, and articulate students. Both continued on to great success after interviewing with us. They excelled in all of their studies, including the two and a half weeks of special relativity covered in the modern physics class they each took (separately). One important difference between the twins though is that Fred's physics classes were inquiry-based, whereas George took traditional classes. Fred's two introductory physics classes and subsequent modern physics class all used Physics Education Research-based Tutorials. On the other hand, George's two introductory physics classes and modern physics class used no such materials.

The twin paradox question

Fred and George eagerly agreed to participate in our “study of student learning of physics,” but neither of them knew ahead of time what we would be doing or what topic we would be talking about. During the interview, they were seated together and asked to resolve a version of the twin paradox problem as we videotaped them. They were given a calculator, some paper, and a modern physics textbook. We encouraged them to work together and openly discuss their disagreements. The brothers expressed themselves clearly and participated equally and enthusiastically in the lively discussion that followed.

The version of the twin paradox problem presented is shown in full in the appendix and summarized in Fig. 1. Fred and George were asked to calculate the amount of time elapsed for the outbound, inbound, and total times of a roundtrip by Bonnie on a spaceship as measured by both herself and by Aaron on Earth. Calculation results are shown in Fig. 2.

To resolve the problem, the students needed to calculate the total time elapsed for Aaron according to Aaron by using the definition of velocity, giving 20 years. They needed to calculate the total time elapsed for Bonnie according to Aaron by application of time dilation, giving 16 years. For Bonnie's point of view, Fred and George needed to recognize that the distance between the planets is smaller due to length contraction and that the time elapsed for Bonnie on her own clock would be 16 years, in agreement with Aaron. We expected that Fred and George would use this information and time dilation to calculate the time elapsed for Aaron according to Bonnie to be 12.8 years (for the two legs). This is different than the time elapsed for Aaron according to himself.

There is therefore an apparent paradox. Each twin (Aaron and Bonnie) correctly calculates a different travel time for Aaron for each leg of the trip. However, when Bonnie turns around at Planet X, she switches reference frames and it changes the symmetry of the situation. Bonnie would observe Aaron's clock running very quickly during her turnaround.
time. Accounting for this asymmetry, the correct conclusion is that Aaron ages 20 years and Bonnie ages 16 years.

We did not expect the twins (Fred and George) to have enough familiarity or experience with special relativity to recognize this asymmetry and to be able to give a complete resolution to this problem. We presented this problem in order to see how they approached the twin paradox (which has no real-world experiences on which to draw) and if their contrasting educational experiences in physics affected how they approached something so different and challenging.

**The twins’ discussion**

Here we describe the process that the brothers went through as they calculated their results. We highlight their different approaches and the reasoning they applied.

Fred and George first worked out the times as calculated by Aaron for the first leg of the trip. They used the definition of velocity to find the time passed for Aaron, and correctly applied time dilation to calculate the time passed for Bonnie. This was a straightforward calculation with which both students were comfortable. They both agreed that according to Aaron for the first leg of the trip, Aaron’s clock should read 10 years and Bonnie’s clock should read eight years.

The next step was for them to find the times for the first leg of the trip as calculated by Bonnie. Before they even tried to calculate it, they argued over what the answer should be. George believed that Bonnie’s results should be the opposite of Aaron’s:

**Fred:** Now we’re changing frames... ... but the answer is the same.

**George:** No!... ... there’s no symmetry.

Fred’s intuition seemed to suggest that the clock reading should be whatever it is regardless of who is reading it. He refers to Aaron and Bonnie calculating the same time. George believed that since Aaron calculated 10 years for himself and eight years for Bonnie, Bonnie should get the opposite: 10 for herself and eight for Aaron. This is what he was referring to by symmetry. In the following excerpt, Fred explained this result:

**Fred:** The fact is that it will take less time for her to reach it. So if we go from his frame it takes 10 years; for her now it should take less because this [Planet X] is closer.

**George:** Well, I know that but, uh, I’m disturbed by it.

**Fred:** You’re disturbed by it?

**George:** I am bothered by it, because I remember working with like the problems about trains and all that. No frame is absolute.

Fred was unable to explain his objections properly to Fred’s explanation. He used the expression “no frame is absolute” to justify his objections, a phrase he may have heard before but applied in the wrong context here.

Fred was eventually able to convince George to calculate the time based on the contracted distance that Bonnie would see and they found the correct result of eight years.

Next, they tried to calculate the time passed for Aaron according to Bonnie. Fred immediately noticed that Aaron’s clock will run slow according to Bonnie and that Bonnie will calculate that the time for him will be 6.4 years. He started to argue that the result of 6.4 didn’t make sense:

**Fred:** So Aaron tells you my clock says 10 and then Bonnie’s goes and she says no: Aaron’s clock says 6.4. How on Earth [author note: no pun apparently intended] can they have two different times if it’s the same, if it’s the same clock? How can they measure two different things?

**George:** It’s the answer.

**Fred:** No! Something is terribly wrong.

Even in this difficult problem, Fred seemed concerned that there was an apparent conflict with real-world expectations when they tried to put all the numbers together. He was confident that they had applied the theory properly to calculate the times for each leg of the trip, but he believed that there must be something missing when they put it all together. On the other hand, George appeared willing to accept the result of the equation.

Fred and George were unable to resolve the difference in calculated times for the outbound leg of the trip. At one point, Fred considered that the difference might be resolved by considering the simultaneity of events:

**Fred:** If you do the coordinate (transformation), the events are reaching the planet and looking at the clock... ... it’s not simultaneous.

Fred approached the problem from a different point of view. He tried thinking about the simultaneity of the events to see if this was relevant. He also demonstrated an understanding of the relativity tutorials that he had been through. Unfortunately this thread of the discussion was brief.

Throughout the discussion, Fred tried to resolve conflicts by discussing why a certain equation was applied and whether or not it was the appropriate approach. George spent more time double checking their calculations and leafing through the modern physics textbook (at one point he reached for the textbook saying, “Let’s put some order in this chaos”). Nevertheless, they were both unable to successfully resolve the paradox.
Near the end of the session, referring to the reunion of Aaron and Bonnie, the interviewer asked them about the conflicting total times:

**Interviewer:** Do these two numbers have to be the same: Aaron’s clock according to Aaron and Aaron’s clock according to Bonnie?

**George:** No.

**Interviewer:** Why no?

**George:** That’s what relativity is.

George eventually came to think that relativity just has this problem built into it. He was willing to accept that there is no resolution and gave up on the possibility that there could be a resolution to this apparent conflict. In contrast, Fred’s responses indicated recognition that when Aaron and Bonnie meet, they need to agree on the times and which of them is older.

Our perceptions of the differences between Fred and George are consistent with our coding of the interview. We developed and implemented a rubric that contrasted positive and negative epistemological approaches. Positive approaches include stressing underlying ideas (“we need to think about what that means”), believing that science is a consistent coherent framework (“why is that answer different than…?”), and working to understand ideas for oneself (“I’m trying to figure out how to…”). Negative approaches include applying equations regardless of whether they are understood (“those variables all fit into…”), believing that science can be treated as unrelated facts (“that’s from a different chapter”), and taking what is given from authority without evaluation (“because that’s what’s in the book”). We counted all statements corresponding to positive and negative epistemological approaches throughout the interview. Fred had a positive to negative ratio of 16 to 1. George had a positive to negative ratio of 9 to 9.

**Conclusions**

We observed two students struggling with the twin paradox problem, which is counterintuitive with respect to space and time and is so difficult that many graduate students have difficulty resolving it. We were able to explore how two students with different physics learning experiences approach trying to solve the problem.

In addition to the twin paradox, we recorded other interviews with Fred and George solving a variety of problems together or individually. They displayed similar approaches that they used for the twin paradox problem as they tried to solve problems in very different ways. George’s approach during the interviews was to look for the correct equation, calculate the result, and then find ways to justify the answer he found. The fact that a result was physically impossible for the twin paradox did not deter him. He assumed it was a result of special relativity. Fred, who in his physics courses was often encouraged to discuss and develop his ideas aloud with his peers, would question any result they found and would try to make sense of it. He also showed greater understanding of underlying concepts. His thinking is more consistent with a student who has a more expert approach to the nature of learning.

The inquiry-based activities Fred went through are designed to help students develop a positive epistemology. This case study of identical twin brothers highlights the impact that inquiry-based activities can have on both student understanding and epistemology.

**References**


3. Fred and George are not their real names and do not necessarily reflect the ethnicity of the twins.


5. Sebastien Cormier, PhD Dissertation, City University of New York, Physics Department (2009).


**Appendix: Twin paradox version studied**

1. **Consider twins** Aaron and Bonnie both at location Earth (see Fig. 1). On the day of their 30th birthday, Aaron is at rest on Earth while Bonnie is on a rocket ship just above Aaron moving 0.6c. Their clocks read the same time at this instant. Bonnie travels directly to Planet X, which Aaron measures to be six light-years from Earth.

   **A.** According to Aaron:
   - How much time passes on his own clock for Bonnie to reach Planet X?
   - How much time passes on Bonnie’s clock for her to reach Planet X?

   **B.** According to Bonnie:
   - How much time passes on her own clock for her to reach Planet X?
   - How much time passes on Aaron’s clock for her to reach Planet X?
C. According to Aaron, when Bonnie reaches Planet X:
– How much total time has passed on his own clock since the beginning?
– How much total time has passed on Bonnie’s clock since the beginning?

D. According to Bonnie, when she reaches Planet X:
– How much total time has passed on her own clock since the beginning?
– How much total time has passed on Aaron’s clock since the beginning?

2. When Bonnie reaches Planet X, she quickly comes to a stop and returns to Earth with the same speed.

A. According to Aaron:
– How much time passes on his own clock during Bonnie’s return trip to Earth?
– How much time passes on Bonnie’s clock during her return trip to Earth?

B. According to Bonnie:
– How much time passes on her own clock during her return trip to Earth?
– How much time passes on Aaron’s clock during her return trip to Earth?

The Simultaneity Spacetime Diagram model in the OSP ComPADRE Collection was written to show the effect of relative motion when observing (recording) events in special relativity. It is a supplemental simulation for the article by Sébastien Cormier and Richard Steinberg. In the default scenario, an explosion (an event) at the center of a right-moving stick occurs at \( t=0 \), and the arrival of the explosion light signal at each end is recorded. The arrival event at the left end occurs before the arrival event at the right end because the stick is moving. How do the location and time of these events change if they are observed in a reference frame (the Other frame) in which the stick is stationary? The Other frame’s spacetime diagram is shown in a second window.

http://www.compadre.org/OSP/items/detail.cfm?ID=10383

Initial conditions, such as the locations of the explosion and the detectors, can be adjusted by dragging the circular markers before the simulation is run. The slider near the top of the main window can be used to change the speed of the stick. A third window shows the stick and the light wave front in space. More importantly, a checkbox allows users to compare the predictions of Galilean and special relativity in order to observe how the assumption of a constant speed of light leads to the relativity of simultaneity.

Related items can be found in the OSP Collection by searching for “special relativity.” In particular, the Twin Paradox program shows a spacetime diagram for a round-trip to a neighboring star.

http://www.compadre.org/osp/items/detail.cfm?ID=7238

This supplemental simulation for the paper by Sébastien Cormier and Richard Steinberg has been approved by the authors and the TPT editor. Partial funding for the development of this model was obtained through NSF grant DUE-0937731.

–Wolfgang Christian