

Vectors - Equilibrium of a Particle

APPARATUS

1. A force table equipped with a ring, pin, four pulleys, cords and pans
2. A set of 16 slotted masses: Set of known masses (slotted type)
(4×100 g, 4×50 g, 2×20 g, 2×10 g, 1×5 g, 2×2 g, 1×1 g)
3. Protractor
4. Ruler

INTRODUCTION

Physical quantities that require both a magnitude and direction for their description are vector quantities. Vectors must be added by special rules that take both parts of the description into account. One method for adding vectors is graphical, constructing a diagram in which the vectors are represented by arrows drawn to scale and oriented with respect to a fixed direction.

Using the graphic method we can rapidly solve problems involving the equilibrium of a particle, in which the vector sum of the forces acting on the particle must be equal to zero. While the graphical method has lower accuracy than analytical methods, it is a way of getting a feel for the relative magnitude and direction of the forces. It can also solve the problem of the ambiguity of the direction of a force where the analytical method uses an arctangent to determine direction, which gives angles in the range -90° to $+90^\circ$.

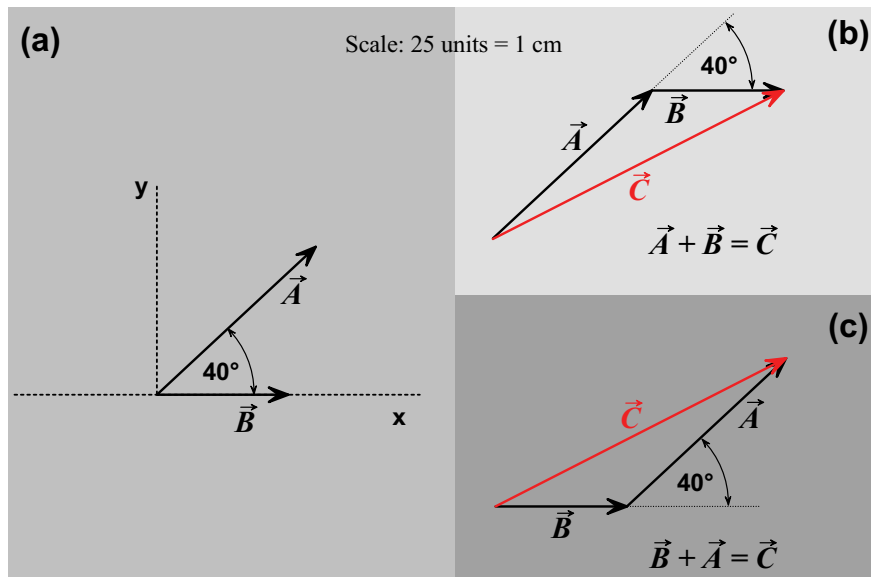


Figure 1: Graphical addition of two vectors

The graphical addition of two vectors is illustrated in Figure 1. A suitable scale must be chosen, so that the diagram will be large enough to fill most of the work space. The scale is written in the work space in the form of an equation, as appears at the top of Figure 1. This is read as "25 units of force are represented by 1 cm on the page".

A 0° direction should also be indicated, for example the x axis in Figure 1(a). Figure 1(b) illustrates starting with \vec{A} , while Figure 1(c) starts with \vec{B} . In each case the second vector is drawn from the head of the first vector (this is the head-to-tail rule). Note that the direction of the second vector is measured from the direction of the first vector as shown in Figure 1(c).

Note also that the vectors are labeled as they are completed.

You can think of this process as "walking" along segments of a path. The equivalent walk, or **resultant vector**, goes from the beginning (tail) of the first vector to the head of the last vector. This is the vector \vec{C} shown in Figures 1(b) and 1(c). The length of this vector can be measured and the scale used to convert back to force units. The direction can also be measured, so that the force vector is then completely described.

In the addition of more than two vectors the procedure is the same, as shown in Figure 2. Vectors are drawn head-to-tail. Shown is $\vec{B} + \vec{A} + \vec{D} = \vec{E}$.

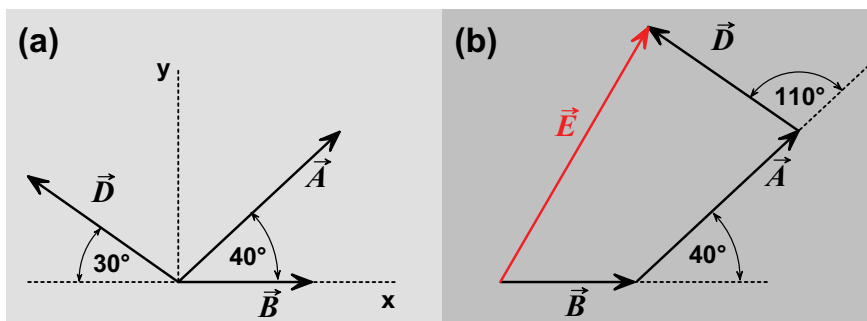


Figure 2: Graphical addition of three vectors

NOTES AND TECHNIQUES

1. We will use "grams" as a unit of force. While not strictly correct, the weight of a given mass is proportional to that mass.
2. The central ring has spokes that connect the inner and outer edges. When connecting the cords be sure that the hooks have not snagged on a spoke. In that case the cord will not be radial. Also whenever you change a pulley position, check that the cord is still radial.
3. Be sure that when you position a pulley, that both edges of the clamp are snugly against the edge of the force table. Check that the cord is on the pulley.
4. There are two tests for equilibrium.
 The first test is to move the pin up and down and observe the ring. If it moves with the pin, the system is NOT in equilibrium and forces need to be adjusted.
 The second test is to remove the pin. However, this should be done in two stages. First just lift it but hold it in the ring to prevent large motions. If there is no motion, remove it completely. If the ring remains centered then the system is in equilibrium.

5. The circle diagram is a quick sketch of the top of the table and summarizes the forces acting on the ring (the particle). Do not spend time making this "pretty". The vector addition diagrams are to be done carefully with ruler and protractor.

PROCEDURE

Part I: Resultant and Equilibrant of Two Forces

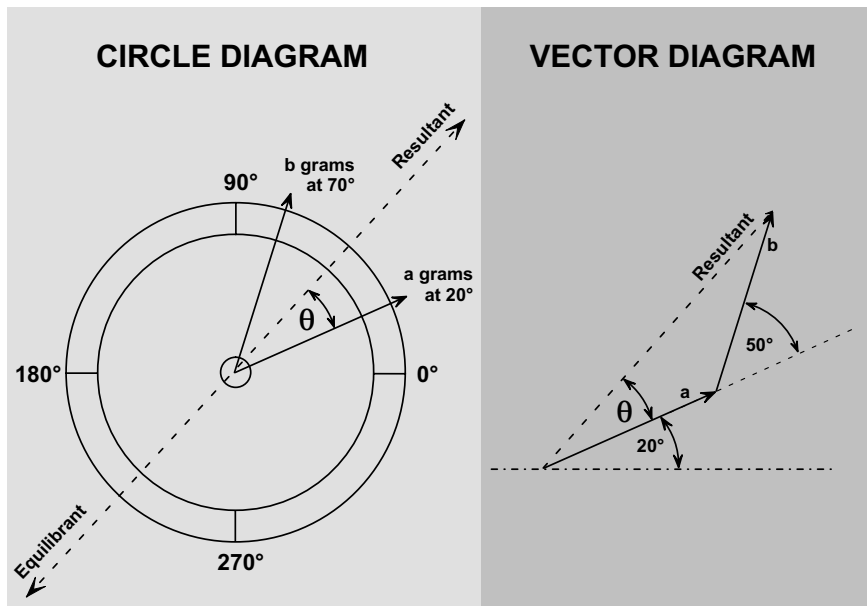


Figure 3

- (a) Mount one pulley on the force table at the 0° mark. Mount a second pulley at any angle between 50° and 80°. Suspend unequal loads on cords running from the central disk over the pulleys. Record the angles and the loads on a circle diagram. Remember that the pans weigh 50 grams.
- (b) Choose a scale so that the sum of the two forces would almost reach across the page. Draw a scale diagram of the two vectors in the "head to tail" position, as in the $\vec{A} - \vec{B}$ portion of Figures 1(c) or 2(b). Complete the diagram so as to determine the magnitude and direction of the resultant, as in Figure 1(c). The resultant is the single force that is the sum of the two original forces.

Write down the size of the resultant and its direction.

- (c) The equilibrant is the single force that would balance the original two forces. It should be equal in size to the resultant, and opposite in direction. Mount a pulley at this angle on the force table, and use a cord to apply the appropriate load. Perform the first test for equilibrium (moving the pin).

You may have to vary the load slightly, or the angle to obtain a good balance. When you think this has been achieved, perform the second test (removing the pin). Record the final values of the load and angle.

Questions (to be answered in your report): How close did the final values come to the values determined from the vector addition diagram? (You should have been able to get within 3 grams and 1.5 degrees.)

Part II: Addition of Four Forces Producing Equilibrium

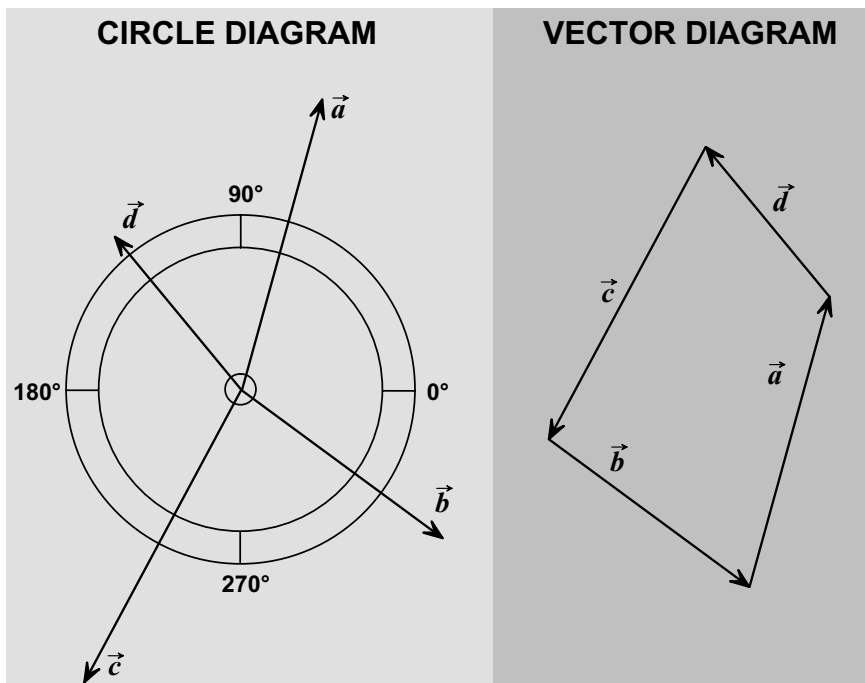


Figure 4

- Set up four pulleys and suspend unequal loads on the cords running over them. Change angles and loads until the system is in equilibrium (i.e. passes both tests). Be sure that the cords are still radial. Sketch the circle diagram to record loads (remember the weight of the pans) and angles.
- Carefully draw the vector addition diagram (see figure 4). Note that your diagram may not close due to small errors.

Questions (to be answered in your report):

- How large is the resultant of the four force vectors in your diagram?
 - Why should we expect the vector addition diagram to close?
- (c) The discrepancy can arise from two sources, errors in the lengths of the lines in our diagram and errors in angles. We can estimate the uncertainty in each by noting the sensitivity of the ruler (ΔL) and protractor ($\Delta\theta$). (Sensitivity is the smallest quantity that can be read

or estimated from a scale.) Convert ΔL to a force by using your scale value. Convert $\Delta\theta$ to radians and multiply by twice the largest force in the diagram. ($180^\circ = \pi$ Rad.) The sum of these two terms is an estimate of the uncertainty in the resultant.

Questions (to be answered in your report): How does the size of your resultant in (b) compare with this uncertainty?

Part III: Determination of X and Y Components of a Force

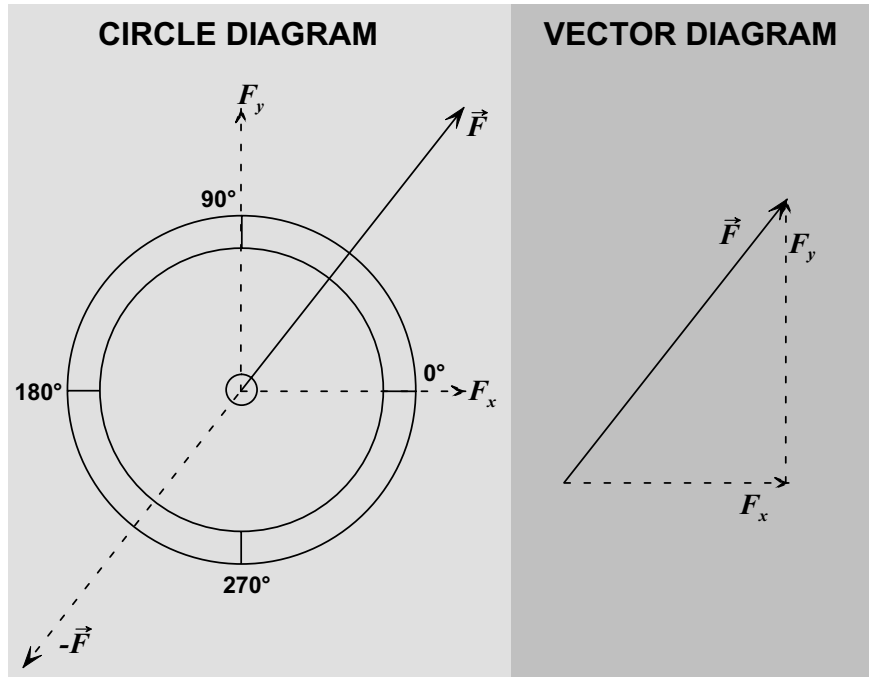


Figure 5

- (a) Place a pulley on the 30° mark of the force table and apply a load over it. Note the total load and angle at the top of a data sheet.
- (b) If the resultant of the vector sum of two forces is the single force that is an exact equivalent of the two original forces, then we can reverse the process and find the two forces, in convenient directions, that is equivalent to any given single force.

Draw a set of X-Y axes at the lower left of your data sheet. Choose a scale so that the vector representing your load will span most of the page. Draw the vector representing your load, assuming that the positive X axis is the 0° direction.

Drop a perpendicular to the X axis from the head of your vector (this line makes an angle of 60° with the direction of the vector).

Draw arrow heads on the two legs of the resulting right triangle, as if they were two vectors that added to the vector on the hypotenuse.

- (c) The vector along the X axis and the vector parallel to the Y axis are called component vectors¹. Convert to force values and sketch a circle diagram with the results. Set up these forces on your force table. Move the original load by 180° , to 210° . Test for equilibrium, and make any necessary adjustments to the load to balance.

Questions (to be answered in your report): : How large an adjustment did you have to make? How does this compare with the uncertainty you found in II.(c)?

- (d) The sides of the right triangle can also be determined by trigonometry. Calculate the size of the component vectors trigonometrically. Show your calculation. How do they compare with the values you found graphically ?

¹Components are scalars that have an accompanying indication of direction.