Elasticity and Simple Harmonic Motion

APPARATUS

1. Balance and set of known masses
2. Two cylindrical springs
3. A set of five masses: 100, 200, 200, 500 and 1000 g (Hook type)
4. Upright meter stick with a movable index attached
5. Stop clock

PROCEDURE

Part I: Elasticity of a body

Since no real body is perfectly rigid the application of a force will distort it. A perfectly elastic body will return to its original form after the removal of the distorting force.

The elasticity of the cylindrical spring may be tested by comparing the position of some point on the spring before each of several loads is added with the position of the point after each load is removed.

Using the vertical meter stick observe the position of some point at the lower end of the spring. Add a load of 400 g and again note the position of the point on the spring. Increase the load by 200 g and repeat the observation. Remove the loads, one at a time, compare the position of the reference point after each load is removed with its corresponding original position.

Part II: Dependence of the distortion on the distorting force

For many bodies the force, within limits, produces a distortion which is proportional to the force. The limits within which this proportionality exists depends upon the material of which the body is made and upon the form of the body.

The body to be studied is a closely wound cylindrical spring. A force large enough to produce a permanent elongation is easily conceivable. Adjacent turns of this spring may press so tightly together that some pull is required to relieve this compressional tendency before the spring can begin to be stretched. There is possible, then, an upper and a lower limit to the force which can be applied to the spring, between which the distortion is proportional to the force.

1. Observe the position of some reference point on the lower end of the spring as in Part I. This is the zero reading.
2. Apply a load of 100 g and note the new position of the reference point.
3. Repeat, adding 100 g each time, until the total load supported is 1000 g.

4. Determine the elongation produced by each load by subtracting the zero reading from each subsequent reading.

5. Show the dependence of the elongation upon the applied force by plotting the elongation as y-axis and the corresponding total force as x-axis.

6. Determine from the curve the range of forces used in which the elongation is proportional to the force.

7. Remove the load 100 g at a time, taking the scale reading in each case. Are these readings the same, for each load, as those found above?

Part III: Force constant of the spring

The force constant of the spring is the force ΔF required to produce an elongation Δl in the spring. In symbols, this may be expressed as $k = \Delta F / \Delta l$ in units of N/m. This is a constant only for the range of forces within which the proportionality of Part II exists.

Determine an average value of the force constant of the spring from the curve plotted in Part II.

Part IV: Dependence of the period in simple harmonic motion on the vibrating mass

Consider a body, for which the distortion is proportional to the force producing it, held away from its normal position. There is now a restoring force in the body, which is proportional to the distortion. If the force applied to the body is removed, this restoring force returns the body to its normal position. However, its inertia carries it through that point producing a distortion or displacement on the other side. Now the action of the restoring force first brings the body to rest in a distorted position. The action is repeated and this simple harmonic motion continues until it is stopped by friction.

As an example, consider as the body the spring with a load of 500 g suspended from it. Reference to the data of Part II will show that now, the spring is in a condition where any additional force will produce a proportional displacement. If the load is pulled down some distance x and released, a restoring force $-k\,x$ acts on the body. As the body moves back to its equilibrium position, this restoring force diminishes. The minus sign indicates that the restoring force is opposite to the distortion. Since $-kx$ is an unbalanced force, it produces an acceleration a. From Newton's Law, $F = M\,a$, we get $-k\,x = M\,a$, where M is the mass of the system and the negative sign shows that x and a are oppositely directed. This yields

$$-\frac{x}{a} = \frac{M}{k} \tag{1}$$

The period T in simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{M}{k}} \tag{2}$$

Here, M is the mass of the vibrating system consisting of the mass suspended from the spring (500 g in the example) plus a part of the mass of the spring. It can be shown that one third of the total mass of the spring is the part effective in determining the total M.

1. Determine the period of the simple harmonic motion occurring when the load on the spring is 500 g. Determine the average time required for at least fifty vibrations.
2. Repeat with loads of 600, 700 and 800 g.
3. Measure the mass of the spring.
4. Using equation (2), calculate the period to be expected in each case. Compare them with the experimental values of the periods. Obtain both, the difference and the percent difference.
5. What percentage error would be introduced in the calculated values of T for the 500 g load and the 800 g load, respectively, if the mass of the spring were neglected?
6. Plot two curves: $T(M)$ and $T^2(M)$ (the period and the square of the period of vibration vs. the mass supported by the spring)

Part V: Dependence of the period on the amplitude

The maximum value of the displacement in simple harmonic motion is the amplitude.

Using a load of not over 600 g, try varying the initial amplitude of the vibration and note the effect on the period. Does the period depend upon the amplitude?