

Buoyancy and Boyle's Law

Part A: Buoyancy and Archimedes' Principle

APPARATUS

1. Electronic balance with stand
2. Beaker
3. Metal object
4. Wooden block
5. Thread
6. Blue liquid

INTRODUCTION

The hydrostatic pressure P at a distance h below the surface of a fluid is given by

$$P = P_0 + \rho g h$$

where P_0 is the pressure at the surface of the fluid and ρ is the density of the fluid.

The hydrostatic pressure exerts a normal force on all surfaces in contact with the fluid. As a result there is a net upward force, called the Buoyant Force F_B , whose magnitude is equal to the weight of the fluid displaced, a relationship known as Archimedes' Principle.

$$F_B = \rho g V$$

where V is the volume of the object below the surface of the fluid. In this experiment, you will weigh objects in air and then measure the effect of submerging them in a fluid. A clearly labeled Free Body Diagram should be used to determine the forces on the submerged objects in order to relate your measurements to the density of the objects. If the fluid is water, assume the standard value for ρ of 1000 kg/m^3 .

The electronic balance is turned on by pressing the button at the right. Pressing the button on the left quickly will change the units displayed. We will work with the gram scale. Note that this is the mass-equivalent of the force being measured, you do not have to actually multiply by the numerical value of g , leave it as a symbol and it will eventually cancel out.

PROCEDURE

Determine the density of a solid more dense than water. Weigh the metal object and then suspend it from the hook on the underside of the balance so that it is submerged in the beaker of water. This second weight, called the "apparent weight" differs from the first due to the buoyant force. Draw the corresponding Free Body Diagram and use it to determine the forces involved, and to solve for the density of the submerged object. Calculate the buoyant force

and the density from your measurements. Use the table of densities on page 5 to estimate the composition of the metal. Does it appear reasonable from the appearance of the material? Explain.

An **alternate procedure** is to place the beaker on top of the scale and measure the change when the object is just submerged while being supported by the thread. Dry the metal object and weigh the beaker before (W_0) and after (W_1) the object is submerged. Do not let the submerged object touch the bottom of the beaker.

What force in the Free Body Diagram does ($W_1 - W_0$) represent? How was this force communicated to the bottom of the beaker? (Hint: what happened to the level of water in the beaker when the object was submerged?)

Determine the density of an object less dense than water. Weigh the wood block in air. Attach the metal object to it so as to act as a "sinker". Use either method to determine the density of the wood block. Explain the procedure that you chose, including appropriate Free Body Diagrams.

Use the table of densities on page 5 to make a guess as to the type of wood provided.

Determine the density of a liquid other than water. You now have objects whose densities are known. One of them can be used as a test object to determine the density of the unknown liquid. Be sure that the object you use is as dry as possible. Use one of the earlier procedures to determine the buoyant force on the object, and calculate the density of the liquid.

Use the table of densities to make a guess as to the composition of the unknown liquid.

Question (to be answered in your report):

How large a mass would have to be placed on top of the wooden block when floating in the water so that the block would be completely submerged, i.e. its top would be level with the surface of the water?

Part B: Boyle's Law

APPARATUS

1. Sealed hypodermic syringe
2. Set of hooked weights
3. Loop of string

INTRODUCTION

Boyle's law states that for a fixed mass of gas at a constant temperature, the product of the absolute pressure p and volume V is a constant:

$$p V = k \tag{1}$$

With a simple apparatus we can hang masses on the syringe plunger, thereby increasing the pressure on the gas inside the syringe to a value above atmospheric pressure. The volume can be read

from the scale on the side of the syringe. The scale is in units of cc which means cm^3 . If the pressure added by the weight of the masses is p_{add} and the atmospheric pressure is p_{atm} , then the resulting total pressure is $p_{total} = p_{atm} + p_{add}$.

PROCEDURE

1. Record the current atmospheric pressure and temperature as indicated by your instructor. (If the current temperature is not available, assume $68^\circ\text{F} = 20^\circ\text{C}$.) Atmospheric pressure may be given in mbar or bar, where $1 \text{ bar} = 10^5 \text{ Pa}$. Convert to Pascals.
- 2a. Check that the syringe is held firmly in the clamp and is vertical. Examine the scale and record the volume V_0 , indicated by the bottom edge of the plunger.
- 2b. Record the sensitivity of the scale, that is, the smallest quantity that can be read or estimated on it. We will use this value as the uncertainty of our volume measurements.
- 2c. How many digits of V_0 are actually significant? (See the discussion on significant figures in the introduction of the lab manual.)
3. We use a string to hang masses on the plunger. The weight of this mass acting on the area of the plunger will increase the pressure of the trapped air and therefore change the volume. We can simplify the calculation of added pressure for each value of mass by considering the pressure p_1 , at a load of 1 kg. There are four calculations:

- (i) the weight of 1 kg, W_1 ,
- (ii) the area of the face of the plunger A , (radius of the plunger = 0.715 cm),
- (iii) the pressure added by the weight, $p_1 = W_1/A$, and
- (iv) conversion of the units to the standard unit of pressure, the Pascal. ($1 \text{ Pa} = 1 \text{ N/m}^2$)

Check : p_1 should be about 3/4 of an atmosphere ($1 \text{ atmosphere} = 1.013 \times 10^5 \text{ Pa}$).

Finally, trim p_1 to the number of significant figures found in 2c. (One extra digit may be kept as a guard digit to avoid round-off problems.)

4. Use the headings below to prepare a table for your data. Note the second line of the heading that contains the multiples and units to be used.

Mass M [kg]	Volume V [10^{-6} m^3]	p_{add} [10^5 Pa]	p_{total} [10^5 Pa]	$1/p_{total}$ [10^{-5} Pa^{-1}]
0	V_0	0	p_{atm}	
...	...			

Your first entry will be for Mass = 0, and Volume = V_0 as found in 2a. Note that the units have to be converted to m^3 ($1 \text{ cm} = 10^{-2} \text{ m} \rightarrow 1 \text{ cm}^3 = 10^{-6} \text{ m}^3$). p_{add} will be zero, and therefore p_{total} is just equal to the atmospheric pressure.

5. Use the string to hang weights on the plunger. Use values of 500, 700, 1000, 1200 and 1500 g. Wait at least a minute after each weight is added, so that the gas can come back to room temperature. Increase the waiting time at the larger loads, so that the gas can return to room temperature after being compressed. Use the time to complete some of the calculations outlined below. Record the volume and complete the line in the table. Remember that $p_{add} = [M/(1 \text{ kg})] \times p_1$, where p_1 is the additional pressure due to a 1-kilogram weight. Trim p_{total} of any digits that are not significant.

ANALYSIS

1. We will use Boyle's law in the form:

$$V = k(1/p) \quad (2)$$

Calculate all the reciprocals and put them into the last column of the table.

- 2a. Plot the graph of V vs. $1/p_{total}$. The origin should be included, although it is not a data point. Be sure that your scales allow the graph to occupy most of the page. (We are following the usual practice of plotting the dependant variable on the y-axis.)
- 2b. Draw the single straight line that best represents the data. Use a transparent straight edge (like a plastic ruler) to help fit the line. The line should be drawn completely across the graph.
- 2c. Choose two points on the line (not data points) that are widely separated to use in calculating the slope. Note that the slope will have units. Record this value as k_{slope} .
3. The graph of equation 2 would go through the origin. With experimental data, the straight line usually comes close to, but misses the origin. Determine the positive intercept with either axis. Assume that the uncertainties in the value of these intercepts are 0.2 cm^3 and $0.04 \times 10^{-5} \text{ Pa}^{-1}$, respectively.

We can see whether this intercept is consistent with 0 by calculating the uncertainty ratio, your intercept value divided by the appropriate uncertainty value. A small ratio (less than 2) indicates good agreement. Large values (greater than 5) means disagreement. Intermediate values can be described as 'fair' or 'poor' agreement and usually require further study.

4. The constant k can also be determined from the Ideal Gas Law, $pV = nRT$, where n is the number of moles of gas ($\rho V_1/MW$), R is the gas constant ($8.31 \text{ J/mol}\cdot\text{K}$) and T is the absolute temperature in Kelvin. V_1 is the volume measured at a load of 1 kg and ρ is the corresponding density. For air, 80% $^{14}\text{N}_2$ and 20% $^{16}\text{O}_2$, the molecular weight $MW = 29 \times 10^{-3} \text{ kg/mol}$, and $\rho = 2.13 \text{ kg/m}^3$ at the 1-kg load and room temperature. Calculate nRT from the data given, and compare with k_{slope} , assuming an uncertainty in k_{slope} of $0.2 \times 10^5 \text{ Pa}\cdot\text{cm}^3$.

Questions (to be answered in your report):

- (a) What curve would equation 1 describe in a graph of p vs. V ?
 (b) How could we graph our data so as to obtain a straight line with slope k ?
- How does your value of k_{slope} agree with nRT ? The uncertainty ratio here is

$$\frac{|k_{slope} - nRT|}{\text{uncertainty in } k_{slope}}$$

- If the uncertainty in the slope is ΔV (from procedure 2b.) divided by the range of your $(1/p)$ values, what is your uncertainty in k_{slope} ?

Density Table

Metal	ρ [g/cm ³]
Aluminum	2.7
Brass (ordinary yellow)	8.40
Bronze - phosphor	8.80
Copper	8.90
Gold	19.3
Iron - wrought	7.85
Iron - gray cast	7.1
Lead	11.3
Steel	7.8
Tungsten	19.3
Zinc - wrought	7.2
Balsa wood (oven dry)	0.11 ... 0.14
Ebony	1.11 ... 1.33
Oak	0.6 ... 0.9
Pine - white (oven dry)	0.35 ... 0.50

Liquid	ρ [g/cm ³]
Alcohol, Methyl	0.80
Carbon Tetrachloride	1.60
Gasoline	0.68
Mercury (20°C)	13.55
Water (0°C)	0.999
Water (4°C)	1.000
Water (15°C)	0.997
Water (100°C)	0.958

Stone	ρ [g/cm ³]
Granite	2.7
Limestone	2.7
Marble	2.6 ... 2.8
Mica schist	2.6
Sandstone	2.1 ... 2.3