The Workshop Project Newsletter

PROGRESSIONS:
PEER-LED TEAM LEARNING

Volume 7, Issue 4

Summer 2006

UMaine Mathematics Education Group
Department of Mathematics and Statistics

THE UNIVERSITY OF MAINE

Workshop 2: Velocity/Rates

A Car Trip With Bob and Sue

References:

Smith, Minton Calculus, 2nd ed, 2002, McGraw Hill, CA, Section 2.1, pgs. 149-160 and Section 2.2, Example 2.11, pg.172

Introduction:

Bob and Sue are sitting around on a hot summer afternoon trying to decide what to do, when Sue mentions that she has been thinking about an activity for next semester's Calculus classes. She explains that it involves a car trip during which odometer and speedometer readings are taken every thirty seconds. "Sounds like fun," says Bob, "Let's go." *Car air conditioning on a hot summer afternoon aside, you are probably thinking, "these two desperately need to get a life*." Sue grabs her laptop, watch and water bottle and heads for the Subaru. *OK this just moved into the category of seriously pathetic.*

The trip odometer is set to zero, and timing begins. Although the path is not a straight line (we are in Maine), we can imagine it as a thread, which we straighten to form single dimension travel. The odometer reading represents the position of the car on the now straight-line path.

1. What does the speedometer reading represent?

I. At the start of the trip, the time is 0.0 minutes and the odometer reading is 0.0 miles. At the end of the trip, the odometer reads 15.0 miles and 23.0 minutes have elapsed.

1. Compute the average trip velocity $\bar{v} = \frac{\Delta s}{\Delta t}$ in miles per minute.

2. If the car maintained a constant speed, and completed the trip in 23.0 minutes, what would the speedometer reading be?

II. **Below is the graph of the position vs. time data points for the car trip.**

1. What would the slope of a secant line through any two of the data points represent?

2. If you drew a smoothed curve through the data points, what physical quantity would the slope of a tangent line to the curve represent?

3. How could you tell from the graph whether the car was moving when timing began?

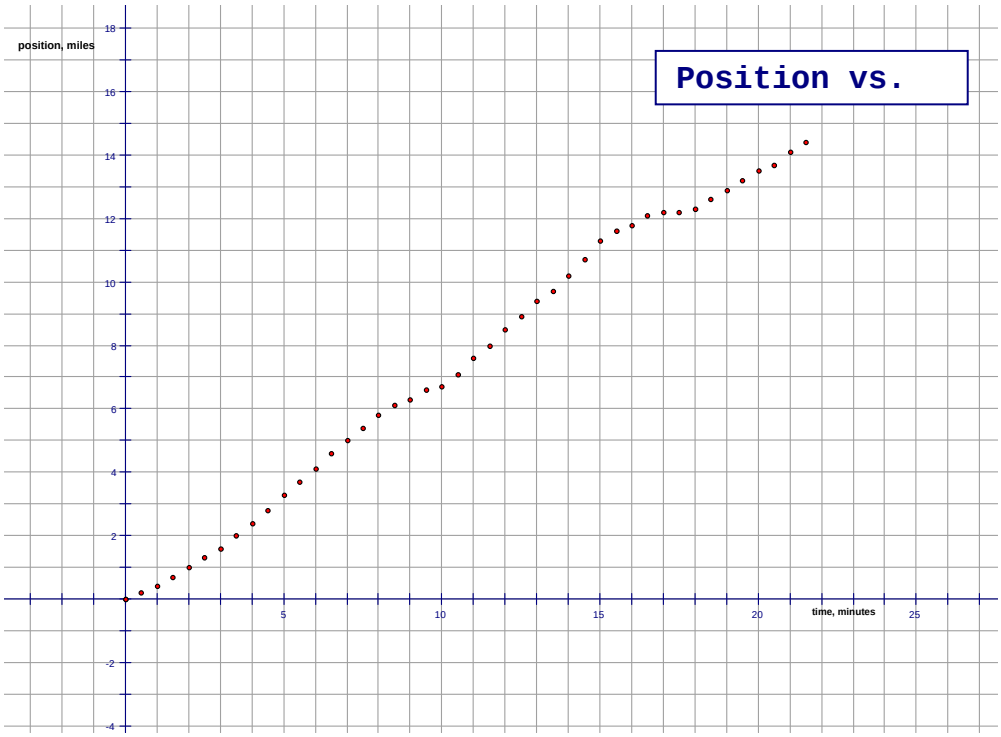
4. Was the car moving when timing began?

5. How could you tell from the graph when the car was moving the fastest?

6. Approximate, from the graph, the time at which the speedometer reading is the highest.

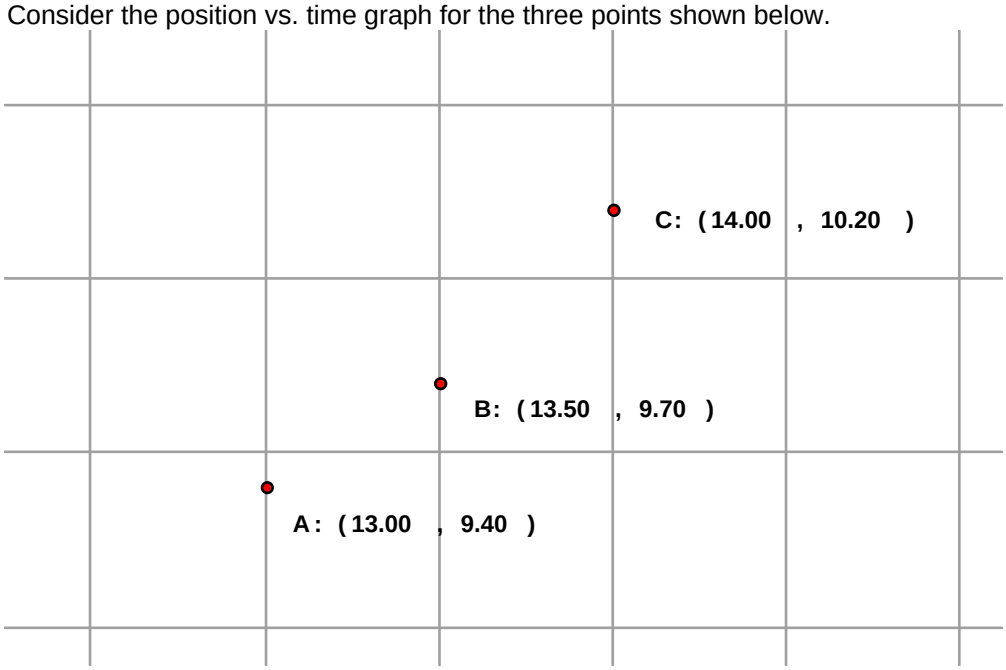
7. What characteristic of the graph would indicate that the car was slowing down?

8. Approximate from the graph, over what time intervals the car is slowing down?



III. A single point of interest.

Consider the position vs. time graph for the three points shown below.



1. Calculate the slope of the secant line through the points A(13.0, 9.4) and B(13.5, 9.7).

2. What does this quantity represent?

3. Calculate the slope of the secant line through the points B(13.5, 9.7) and C(14, 10.2).

4. What does this quantity represent?

5. Calculate the average of these two slopes.

6. We will use the average of the two slopes to approximate the slope of the tangent at the point B(13.5, 9.7). Determine the equation of the line through the point (13.5, 9.7) with this slope.

IV. The table on the attached sheet contains the odometer readings for the trip.

Using the procedure developed in part III, we compute the average velocity between each pair of data points. From this we approximate the instantaneous velocity at each time step.

1. In the column "Change in s" calculate the difference in position between the two data points. For example from (0.5 min, 0.2 miles) to (1.0 min, 0.4 miles), the change in position is 0.4 miles – 0.2 miles = 0.2 miles.

2. In the column "Ave. v", divide the change in s by the change in t, (.5 min.). Continuing with our example the average velocity from 0.5 min to 1.0 min. is:

$$\frac{Change\ in\ s}{Change\ in\ t} = \frac{0.2 miles}{0.5 \min utes}$$
$$= 0.4 \frac{miles}{\min ute}$$

3. In the column "Approx. v," compute the average of the Ave. v for the given data point and the one following it. If we want the approximate velocity at the instant where t = 0.5 minutes, we average the two average velocities: 0.4 mi/min from 0.0 to 0.5 min, and 0.4 miles/min. from 0.5 to 1.0 min. The approximate velocity at t = 0.5 minutes is therefore 0.4 miles/min.

   Explain what this quantity represents.

Odometer Readings

Time (min.)	Position (miles)	Change In s (mi)	Ave. v (mi/min)	Approx.v (mi/min)	Time (min.)	Position (miles)	Change In s (mi)	Ave. v (mi/min)	Approx.v (mi/min)
0.0	0.0				**11.5**	8.0			
		0.2	0.4				0.5	1.0	
0.5	0.2			0.4	**12.0**	8.5			0.9
		0.2	0.4				0.4	0.8	
1.0	0.4			0.5	**12.5**	8.9			
		0.3	0.6				0.5	1.0	
1.5	0.7			0.6	**13.0**	9.4			
		0.3	0.6				0.3		
2.0	1.0			0.6	**13.5**	9.7			
		0.3	0.6				0.5		
2.5	1.3			0.6	**14.0**	10.2			
		0.3	0.6				0.5		
3.0	1.6			0.7	**14.5**	10.7			
		0.4	0.8				0.6		
3.5	2.0			0.8	**15.0**	11.3			
		0.4	0.8				0.3		
4.0	2.4			0.8	**15.5**	11.6			
		0.4	0.8						
4.5	2.8			0.9	**16.0**	11.8			
		0.5	1.0						
5.0	3.3			0.9	**16.5**	12.1			
		0.4	0.8						
5.5	3.7			0.8	**17.0**	12.2			
		0.4	0.8						
6.0	4.1			0.9	**17.5**	12.2			
		0.5	1.0						
6.5	4.6			0.9	**18.0**	12.3			
		0.4	0.8						
7.0	5.0			0.8	**18.5**	12.6			
		0.4	0.8						
7.5	5.4			0.8	**19.0**	12.9			
		0.4	0.8						
8.0	5.8			0.7	**19.5**	13.2			
		0.3	0.6						
8.5	6.1			0.5	**20.0**	13.5			
		0.2	0.4						
9.0	6.3			0.5	**20.5**	13.7			
		0.3	0.6						
9.5	6.6			0.4	**21.0**	14.1			
		0.1	0.2						
10.0	6.7			0.5	**21.5**	14.4			
		0.4	0.8						
10.5	7.1			0.9	**22.0**	14.6			
		0.5	1.0						
11.0	7.6			0.9	**22.5**	14.8			
		0.4	0.8						
11.5	8.0				**23.0**	15.0			

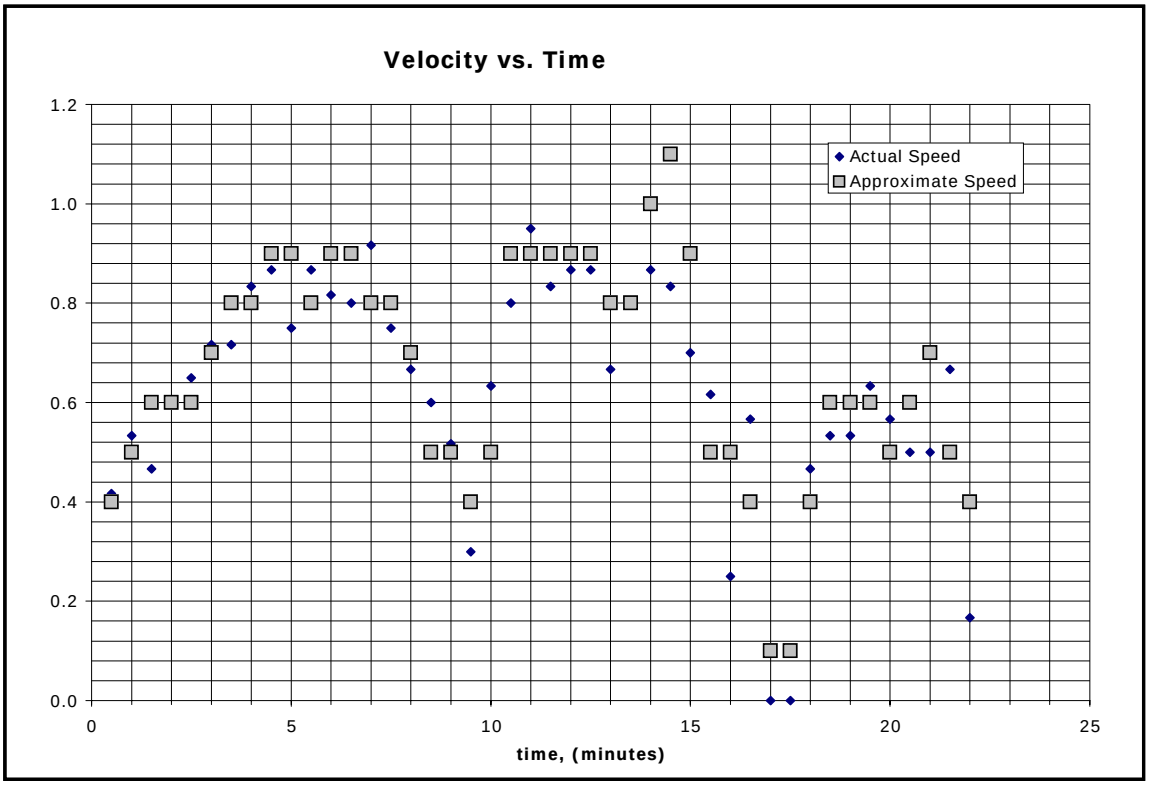
V. The graph below presents the speedometer readings as a function of time for the trip, after they have been converted to miles per minute. On the same graph, are the approximate velocities computed from part IV:

1. Comment on how well the computed velocity matches the speedometer readings.

2. Do you think that the method we used to compute the approximate velocities, that is also employed by your text, is a good way to approximate instantaneous rates of change for discrete data sets?

3. On the graph of velocity vs. time, graph the line: $v = \bar{v}$, (average velocity) using the quantity you computed in part I.

4. This line represents the average velocity over the whole trip. Is this line consistent with the speedometer readings, and/or the computed velocities?



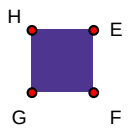
VI. In parts I to V we explored the idea that given information about the position of a car at time, t, we can find the instantaneous rate at which that position is changing (velocity). We compared our calculations or approximate v's with the actual speedometer readings.
The second big idea of Calculus is that given the rate of change of position (velocity), we can determine the net change in position by finding the area between the velocity vs. time graph and the t - axis.

On the graph below, the odometer readings in miles, and speedometer readings, in miles per minute, are given for the trip starting at time, t = 0, and ending with t = 21.5 minutes.

1. How far has the car traveled in this time frame according to the odometer reading?

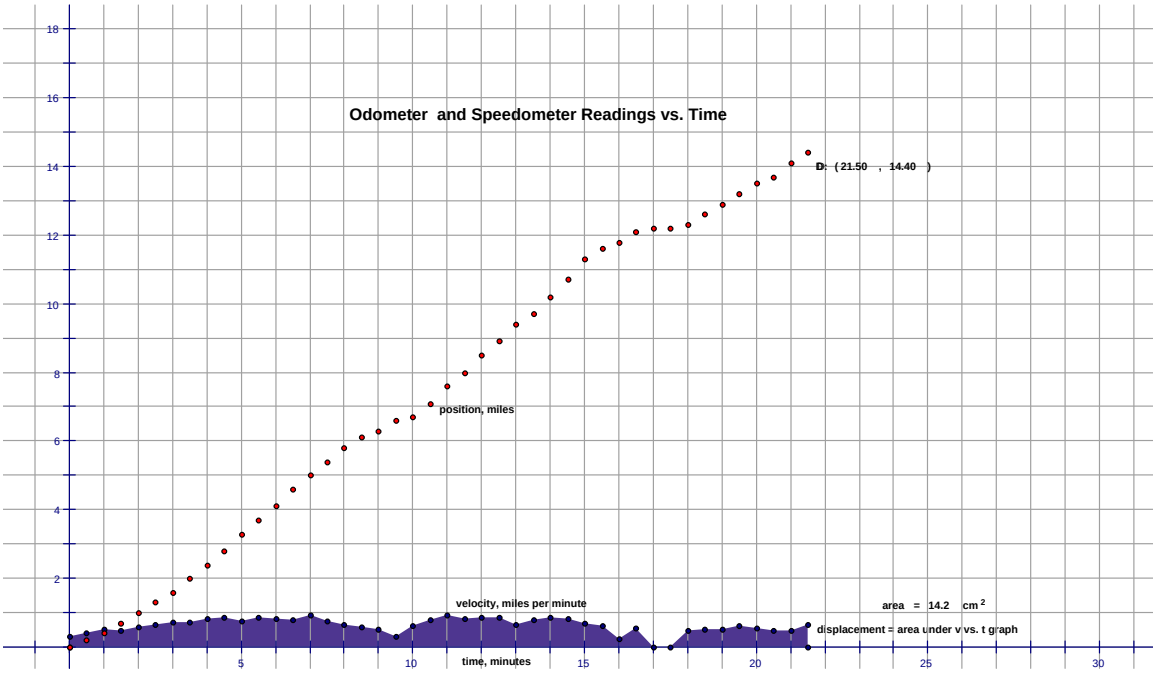
2. Consider the velocity graph. The actual area of a 1mile/min X 1 min. square on the graph is 1.04 cm^2. The measured area under the velocity graph is 14.2 cm^2. This area represents the displacement of the car from 0 to 21.5 min. Compute this displacement in miles.

Area EFGH = 1.04 cm^2



3. How does this compare with the ending odometer reading?

4. If you were to graph the horizontal line representing the average velocity for the trip, and compute the area of the rectangle formed by this line and the t – axis between 0 and 21.5 minutes, what would you expect the area to be?



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