The Workshop Project Newsletter

PROGRESSIONS:
PEER-LED TEAM LEARNING

Volume 7, Issue 2

Winter 2006

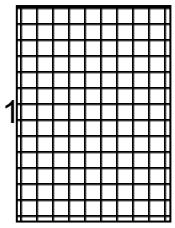
Module 9: Quadratic Equations

Objectives

- To use geometric area models to understand and solve quadratic equations
- To solve quadratic equations using the Pythagorean Theorem

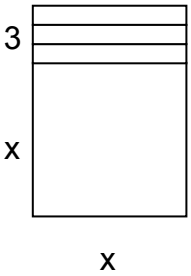
Module 9A: Pre-Lecture

Geometric area models are often used to promote understanding of multiplication concepts in arithmetic. These same models can be used to help understand and solve quadratic equations in algebra. Study the models from arithmetic and algebra below.



Given: The base of the rectangle is 7 units.
The height is 10 units.

Area = base * height = 7 * 10 = 70 square units



Given: The area of the rectangle is 70 square units.
The base is x units. The height is x + 3 units.

Area = base * height = x(x + 3) = 70 square units
or $x^2 + 3x = 70$ square units

When we compare the two models, it is obvious that x = 7 in the quadratic equation $x(x + 3) = 70$ or its equivalent $x^2 + 3x = 70$. So the positive root of the quadratic can actually be found by inspection. Clearly, only a knowledge of algebraic multiplication of integers will help us understand that there is also a negative root, x = -10, because 7 * 10 = 70 and -10 * -7 = 70. Remember that dimensions of plane geometric figures cannot be negative. For certain quadratics, we can now see that both the positive and negative roots can easily be found. We can use the combination of a knowledge of arithmetic, geometry and algebra to do it.
There are several algebraic solutions for the quadratic equation. The following solution is based on factoring and the Zero Product Property:

$$x(x + 3) = 70$$

$$x^2 + 3x = 70$$

$$x^2 + 3x – 70 = 70 – 70$$

$$x^2 + 3x – 70 = 0$$

$$(x - 7)(x + 10) = 0$$

$x – 7 = 0$ $\qquad$ $x + 10 = 0$

$x = 7$ $\qquad$ $x = -10$

We can also show that each one of these answers will algebraically check:

Check:	For x =7	For x = -10
	x(x + 3) = 70	x(x + 3) = 70
	7(7 + 3) = 70	-10(-10 + 3) = 70
	7(10) = 70	-10 (-7) = 70
	70 =70 √	70 = 70√

Use the preceding information as a guide. For each of the following problems, sketch an area model and mentally find the positive and negative roots of the quadratic equation. Then solve the problem and check the answers algebraically.

1. $x(x + 4) = 60$

2. $(x + 1)(x + 4) = 54$

3. $(x+7)(x+6) = 56$

4. $(x + 2)^2 = 25$

5. $(x - 3)^2 = 36$

6. $x^2 + x = 6$

7. $2x(x + 5) = 72$

8. $3x^2 + 6x = 24$

9. Four units have been added to the base and the height of a square. The figure now contains 196 square units. What were the dimensions of the original square?

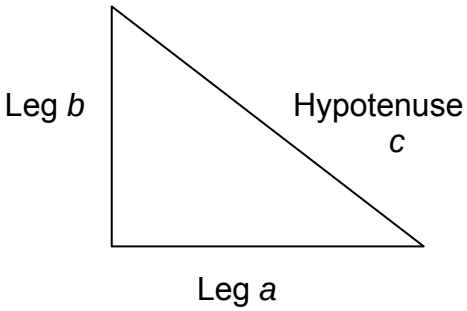
10. A square piece of carpeting has 3 feet trimmed from one side and 6 feet trimmed from the other side to fit the floor of a room containing 108 square feet. What was the area of the original carpet?

Module 9B: Post-Lecture

The Pythagorean Theorem: In a right triangle (a triangle that has a 90° angle), the sum of the squares of the lengths of both legs is equal to the square of the length of the hypotenuse. Symbolically,

$$a^2 + b^2 = c^2$$

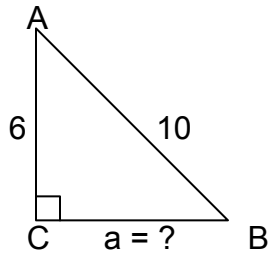
$$(leg)^2 + (leg)^2 = (hypotenuse)^2$$



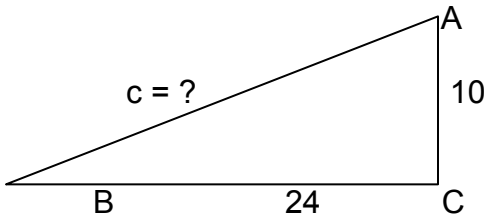
Use the Pythagorean Theorem to solve the following problems:

Solve.

1.



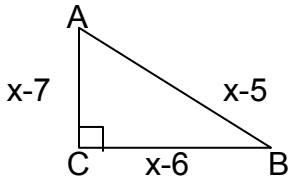
2.



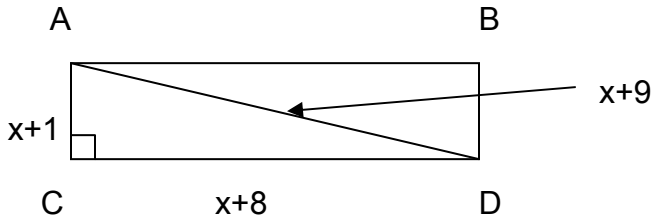
3. An electrician needs to run a cable from the top of a 48-foot tower to a transmitter box located 20 feet away from the base of the tower. Find how long she should cut the cable.

4. The shorter leg of a right triangle is 3 meters less than the other leg. Find the length of the two legs is the hypotenuse is 15 meters.

5. Find x, the dimensions of the right triangle, and the area of the right triangle.



6. Find x, the dimensions of the rectangle, and the area of the rectangle.



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David Gosser, Consulting Editor, gosser@sci.ccny.cuny.edu
AE Dreyfuss, Editor, aedreyfuss@ccny.cuny.edu
Janet Liou-Mark, Co-Editor, Algebra Modules,
JLiou-Mark@CityTech.cuny.edu
June Gaston, Co-Editor, Algebra Modules, jlg1196@aol.com

The City College of New York
Marshak Science Building, MR-1024
160 Convent Avenue, New York NY 10031
Phone: 212-650-5704 Fax: 212-650-8339
Email: info@pltl.org Website: www.pltl.org

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