

PROGRESSIONS:
PEER-LED TEAM LEARNING

Module 2: Irrational Numbers

Objective

- ❖ To introduce irrational numbers

Module 2A: Pre-Lecture

1. Use a calculator to determine the square roots you do not know. Write the answer your calculator indicates. Do not round off.

$$\begin{aligned}\sqrt{0} &= \underline{\hspace{10cm}} \\ \sqrt{1} &= \underline{\hspace{10cm}} \\ \sqrt{2} &= \underline{\hspace{10cm}} \\ \sqrt{3} &= \underline{\hspace{10cm}} \\ \sqrt{4} &= \underline{\hspace{10cm}} \\ \sqrt{5} &= \underline{\hspace{10cm}} \\ \sqrt{6} &= \underline{\hspace{10cm}} \\ \sqrt{7} &= \underline{\hspace{10cm}} \\ \sqrt{8} &= \underline{\hspace{10cm}} \\ \sqrt{9} &= \underline{\hspace{10cm}} \\ \sqrt{10} &= \underline{\hspace{10cm}}\end{aligned}$$

2. Four of the above answers are integers. These are the answers that you probably calculated mentally. Circle those square roots with their corresponding integers. Recall that integers are RATIONAL NUMBERS, so the corresponding square roots would also be classified as RATIONAL NUMBERS.

The other answers are not integers. They represent examples of IRRATIONAL NUMBERS. The long decimal strings associated with irrational numbers do not really end; the calculator does not have enough display space to show more of the decimal so

it rounds off. These are called non-terminating decimals because they do not end. Notice that the digits in the decimals do not repeat; these are non-repeating decimals. When a square root has a corresponding decimal value that is both non-terminating and non-repeating, it is an IRRATIONAL NUMBER.

We can see that some square roots represent rational numbers and other square roots represent irrational numbers. Without using your calculator, classify the following square roots as rational or irrational.

$\sqrt{9}$	_____
$\sqrt{10}$	_____
$\sqrt{15}$	_____
$\sqrt{16}$	_____
$\sqrt{24}$	_____
$\sqrt{25}$	_____

3. Clearly it is easier to work with square roots that are irrational than with their corresponding non-terminating, non-repeating decimals. Note that if we round off the decimals, we actually change the value of the number. We can rewrite square roots without changing the value of the number. To do this, we need to be familiar with the square roots of perfect squares:

$$\sqrt{1} = 1 \quad \sqrt{4} = 2 \quad \sqrt{9} = 3 \quad \sqrt{16} = 4 \quad \sqrt{25} = 5$$

List all of the remaining square roots of perfect squares up to and including 100.

4. To rewrite or simplify irrational square roots, we try to find the LARGEST perfect square factor of the radicand (number under the radical). Study the following example:

Example:

$$\begin{aligned} \sqrt{200} &= \sqrt{(100)(2)} \\ &= \sqrt{100} \cdot \sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

Note: 100 is the largest perfect square factor of 200.

$$\text{Note: } \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Using a calculator, it can be shown that $\sqrt{200} = 10\sqrt{2}$ because they have the same decimal value:

$$\sqrt{200} = 14.142135623730950488016887242097$$

$$10\sqrt{2} = 14.142135623730950488016887242097$$

Follow the model to simplify each of the following irrational roots and check your answers.

$\sqrt{48}$	$\sqrt{72}$	$\sqrt{180}$	$\sqrt{640}$
Check:	Check:	Check:	Check:

5. Use your number sense to ESTIMATE the decimal value to the nearest tenth. Use your calculator to check how close your estimate is.

	Estimate	Actual
$\sqrt{57}$		
$\sqrt{79}$		
$\sqrt{95}$		

*Progressions: Peer-Led Team Learning
The Workshop Project Newsletter
Winter 2006, Volume 7, Issue 2*

David Gosser, Consulting Editor, gosser@sci.ccny.cuny.edu

AE Dreyfuss, Editor, aedreyfuss@ccny.cuny.edu

Janet Liou-Mark, Co-Editor, Algebra Modules,

JLiou-Mark@CityTech.cuny.edu

June Gaston, Co-Editor, Algebra Modules, jlg1196@aol.com

The City College of New York
Marshak Science Building, MR-1024
160 Convent Avenue, New York NY 10031
Phone: 212-650-5704 Fax: 212-650-8339
Email: info@pltl.org Website: www.pltl.org

Reproduction of material appearing in *Progressions* is encouraged with complete citation. *Progressions* (ISSN 1539-1752—print; ISSN 1539-7483—online) is published by the PLTL Workshop Project.

This newsletter is supported by a grant from the National Science Foundation's Division of Undergraduate Education. The views expressed herein do not necessarily represent those of the National Science Foundation.