

PROGRESSIONS:
PEER-LED TEAM LEARNING

Module 1: Sets

Objective

- ❖ To introduce set theory

Module 1A: Pre-Lecture

The concept of a *set* is important in the study of mathematics. **A set is a collection of objects that is so clearly described that it is always possible to establish whether an object belongs to it. Each object belonging to a set is called a *member* or *element* of the set.**

Braces { } are used to indicate sets. Note that braces are not the same as parentheses () and brackets []. The symbol \in means *is an element of*. The symbol \notin means *is not an element of*.

Given a set of natural numbers less than 25.

This set may be written in different ways, for example, using:

1. **a list or roster:** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24}

Note that using a letter to represent the list or roster of numbers, we write:

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$$

This is read *A is the set whose elements are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, and 24.*

Because this set has so many elements, it can be written more easily, $A = \{1, 2, 3, \dots, 24\}$

This is read *A is the set whose elements are 1, 2, 3, and so on to 24.*

2. a description:

$$A = \{\text{natural numbers less than 25}\}$$

This is read *A is the set whose elements are natural numbers less than 25.*

3. set-builder notation:

$$A = \{x \mid x < 25 \text{ and } x \text{ is a natural number}\}$$

This is read *A is the set of all elements x such that x is a natural number less than 25.*

Note that there are also three ways to write an infinite set of elements.

For example, the set of all natural numbers can be written using

1. the listing or roster method: $N = \{1, 2, 3, \dots\}$
2. a simple description: $N = \{\text{natural numbers}\}$
3. set-builder notation: $N = \{x \mid x \text{ is a natural number}\}$

It is now possible to illustrate how set notation may be used to show that

2, 9, 17, and 21 are members of set A and 0, 9.5, 17/2, and $\sqrt{5}$ are not members of set A:

$$2 \in A$$

$$0 \notin A$$

$$9 \in A$$

$$9.5 \notin A$$

$$17 \in A$$

$$17/2 \notin A$$

$$21 \in A$$

$$\sqrt{5} \notin A$$

In the space below, use set N to show that:

50, 125, 5678 and 12345 are members of set N and 0, 1/5, 5.678 and $\sqrt{7}$ are not members of set N.

Given a set of numbers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

1. Write the set of numbers in three different ways.

2. Name the given set T. Write ten examples including five that show certain elements are members of set T and five that show certain elements are not members of set T.

Given the set of all multiples of 5 that are natural numbers.

3. Write this set of multiples of 5 in three different ways.

4. Name the given set F. Write ten examples including five that show certain elements are members of set F and five that show certain elements are not members of set F.

Given a set: New York, Los Angeles, Chicago, Houston, Philadelphia, Phoenix, San Diego, Dallas, San Antonio

5. Write the set in three different ways.

6. Name the given set C. Write ten examples including five that show certain elements are members of set C and five that show certain elements are not members of set C.

Given a set of objects: apple, cherry, cranberry, grape, strawberry, tomato, watermelon.

7. Write this set in three different ways.

8. Name the given set S . Write ten examples including five that show certain elements are members of set S and five that show certain elements are not members of set S .

Module 1B: Post-Lecture

Write the answers to the following questions on a separate sheet of paper.

1. Give three examples of finite sets.
2. Give three examples of infinite sets.
3. If $A = \{a, b, c, \dots, j\}$ then $n(A) = ?$
4. If $B = \{x \mid x \text{ is a positive even integer between 10 and 20}\}$ then $n(B) = ?$
5. If $C = \{x \mid x \text{ is an odd integer between 7 and 9}\}$ then $n(C) = ?$
6. Give an example of two sets that are equivalent, but not equal.
7. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{2, 4, 6\}$. Find
 - a. The complement of set A
 - b. $(A' \cup B) \cap C$
 - c. $A - B$
 - d. $A - (B \cup C)'$
 - e. The number of proper subsets of U

8. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{2, 4, 6\}$. Determine whether each statement is true or false.
- $3 \notin C$
 - $A \subseteq B$
 - $n(A \times B) = \{4\}$
 - $A \in U$
 - $A \sim C$
9. Use a Venn Diagram to show that $(A \cup B)' \neq A' \cup B'$
10. Show that $n(\{\text{positive odd integers}\}) = \aleph_0$
11. Name two mathematicians who contributed to the development of set theory and briefly describe the contribution of each.

Given the following results of a survey of students who had seen films with military themes:

20 had seen Saving Private Ryan
 15 had seen Platoon
 25 had seen Pearl Harbor

5 had seen Saving Private Ryan and Platoon
 8 had seen Saving Private Ryan and Pearl Harbor
 6 had seen Platoon and Pearl Harbor

4 had seen all three films
 5 had not seen any

- Organize the given information by drawing a Venn Diagram and calculating the set cardinalities.
- How many students saw exactly two of these movies?
- How many students saw Saving Private Ryan, but neither of the other movies?
- How many students were surveyed?

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