The q-theory of finite semigroups: Errata, solved problems and supplements

John Rhodes Benjamin Steinberg

August 5, 2009

Errata

- Page 83, Question 2.4.10: The last line should be changed to "Is the converse always valid when **V** is not a pseudovariety of groups?"
- Page 470, last line: $S_{ab\neq 0}$ should be replaced by R_4 where R_4 is the four element semigroup with zero presented by $\langle a, b \mid ab = ba, a^2 = 0 = b^2 \rangle$.
- Page 482, line after Question 7.3.5: "It turns out" instead of "In turns out."
- Page 487, Question 7.3.14: The statement "One needs at least that the permutation group is subdirectly indecomposable as a permutation group" is incorrect.
- Page 615, Problem 1: The last line should be changed to "Is the converse always valid when **V** is not a pseudovariety of groups?"
- Page 615, Problem 2: The last line should be changed to "Does it hold when **V** is not a pseudovariety of groups?"
- Page 618, Problem 39: The statement "One needs at least that the permutation group is subdirectly indecomposable as a permutation group" is incorrect.

Solved problems

- Karl Auinger has solved Problem 5 in the negative. He has constructed a decidable pseudovariety of groups **H** so that **gDH** is undecidable.
- We have solved Problem 13. For $n \geq 1$, let V_n be the pseudovariety of relational morphisms defined by the relational pseudoidentity

$$e_n = (\{x\}, x^{n+1} = x, \{x^{n+1} = x\})$$

and $\mathsf{V} = \bigcap \mathsf{V}_n$; the V_n are positive as is V. Let $\alpha_n = \mathsf{V}_n\mathfrak{q}$ and put $\alpha = \mathsf{V}\mathfrak{q} = \bigwedge \alpha_n$. Then $\alpha(\mathbf{G}) = \mathbf{CR}$, whereas $B_2 \in \bigcap \alpha_n(\mathbf{G})$ and so the meet of the α_n is not pointwise. Indeed, suppose $S \in \alpha(\mathbf{G})$ and $\varphi\colon S \to G$ is a relational morphism with $\varphi \in \mathsf{V}$, $G \in \mathbf{G}$ and $|G| = n$. Then the fact that $\varphi \models e_n$ immediately yields $S \in \mathbf{CR}$. Conversely, if $S \in \mathbf{CR}$, then the collapsing morphism $S \to \{1\}$ shows that $S \in \alpha(\mathbf{G})$. On the other hand, let $n \geq 1$ and let p be a prime greater than n. Define a relational morphism $\varphi\colon B_2 \to \mathbb{Z}_p$ by $(1,2)\varphi = 1$, $(2,1)\varphi = -1$, $(i,i)\varphi = 0$ (for $i = 1, 2$) and $0\varphi = \mathbb{Z}_p$. Then it is straightforward to verify that φ is a relational morphism satisfying e_n. Thus $B_2 \in \alpha_n(\mathbf{G})$ for all $n \geq 1$.

- Unbeknownst to the authors, Yevhen Zelenyuk had already solved Problem 16 before the book was completed. More precisely, he gave a construction of all the finite projective semigroups in:

  Y. Zelenyuk. Weak projectives of finite semigroups. *Journal of Algebra*, 266(1):77–86, 2003.

  From his construction, decidability is clear.

- We can show that in Problem 39 when $(\mathbf{n}, G)$ is the regular representation of a cyclic group of prime power order, then $S_{(\mathbf{n},G)}$ is fji and the associated exclusion variety is $\mathbb{L}\mathbf{G} ⓜ \mathsf{Excl}(G)$.

Supplements

- Supplement to Section 7.3: If S is an fji semigroup, then $S^{\bullet}$ is also an fji semigroup and $\mathsf{Excl}(S^{\bullet}) = \mathbb{L}\mathsf{Excl}(S)$.